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A Metacognitive Approach to Solving Algebra Problems

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Abstract

The objective of this study is to analyse students' metacognitive approach to the solution of algebra problems. The subject was the students at the State Junior High School 10 Malang, Indonesia, consisting of 43 and 6 students for quantitative and qualitative studies through paired interviews, respectively. The subjects for interviews were chosen on the basis of their learning achievement of which three pairs of students with high, medium and low achievements each. The results of the quantitative study showed that the cognitive approach to the solution of algebra problems, consisting of four aspects: awareness, cognitive strategies, planning and reviewing were in average 2.91; 2.98; 2.83 and 2.85 respectively, and the average as a whole was 2.89. Moreover, these results were supported by those from the interviewes, namely that planning and review aspects were given less attention and that the highest attention was on the cognitive strategy aspect in the solution of algebra problems. This study contributes in the literature by identifying studying stages of metacognitive treatment as a principle adopted in research and and being complemented with interviews to complete the results of the sudy on the stages in students' metacognitivr study in solving algebra problems.

Keywords: metacognitive, problem solving, algebra.

1. Introduction

Each activity done must have either positive or negative impact. One of the negative impacts is that it may result in problems, and each problem should be solved. Solving a problem needs science and knowledge in accordance with the types and levels and an information is received instead of memorized. The received information, however, is a part of ways to solve the problem (Akhsanul, 2010).

From a process perspective, learning means relating the schemata possessed to the new schemata acquired from learning as a combination between the two schemata. The combined schemata as a base for solving problems in accordance with the level and type. Through such a proces, learning not only change and repair knowledge, skills, attitudes, and thinking abilities into the better, but also the more meaningfull, instead of merely collecting a number of facts without any understanding of their meanings.

Some studies had a high percentage of failures and a majority of learners reached a medium level of failures in the field of mathematics (Saad, 2004). The studies are in line with some apprehension some mathematicians have about the ways to solve mathematic problems, but most educators realize that there are a lot of students who have difficulties in solving the problems (Saad, 2004). Solution of mathematic problems is an abstract and difficult process since it involves thoughts and reasoning power (NCTM, 2000).

Algebra, one of studies in mathematics, shows that difficulties and an algebra approach are caused by arithmetic instructions focused on any decision of algebraic processes compared with the structured aspect. The learning of algebra is focused on a representative of symbol produced through a generalization process by mathematizing various conditions (Sfard, 1994). Some studies made so far in the field of algebra have been focused on problems solvings dealing with to strategies and concepts where they suggest that teachers implement more effective learnings (Saad, 2004). An aspect that should be focused on in the learning of algebra is the students' ways of thinking when solving problems in order to understand the students' mathematic thinking processes.

Students' mathematics thinking procesess has not been well considered, but these play a very meaningful role in solving mathematic problems, it is called a metacognitive approach. Therefore, an emphasis should be given on the implementation of mathematic teaching focused on students' thinking process and it may be implemented though a metacognitive appeoach.

In order to be able to have an appropriate solution to a complicated problem, a various types of metacognitive process is needed (Michael, 2006). A successfull problem solver is aware that he himself may lead his effort to look for and to identify any previously bellitled ways when combining information and relationship between previous knowledge and the condition of the existing problem. A less experienced student cannot monitor his process in solving problems effectively and may always adopt unsuccessful strategies (Lerch, 2004). According to O'Neil and Abedi (1996) a metacognitive approach consists of four aspects, namely (1) awarenes; (2) cognitive strategy; (3) planning and (4) review.

Based on the understanding of the four aspects and the application to the implementation of an activity or the solution of a problem, such an implementation or solution will be successfull in accordance with the predetermined planning.

2. Review of Literature

Metacognitiion involves one's knowledge and awareness of his own cognitive activities or something dealing with one's cognitive activities (Livingston, 1997; Schoenfeld, 1992). As a result, one's cognitive activities such as planning, monitoring, and evaluation of the completion of a certain task is a pre-determined cognition (Livingston, 1997). Meanwhile Mohamad (2000) states that metacognitive action possesses two components, namely (1) knowledge of cognition , and (2) a mechanism of self-control and cognitive monitoring. Livingston (1997) showes that metacognitive covers two components, (1) cognitive knowledge; and experiences or metacognitive regulations. The same opinions are also proposed by Huitt (1997) that there are two components included into metacognitive: (1) what we know or what we don't know; and (2) regulation on how we learn. Desoete (2001) states that metacognitive has three components in solving mathematic problems in teaching and learning, they are (1) cognitive knowledge; (2) cognitive skill, and (3) cognitive trust.

Some expters describe cognitive roles to understand and to solve problems. Zimmerman and Rappaport (1988) metacognitive strategies involve thoughts, determine gols in teaching, plan, monitor the implementation process and evaluate each implementation that should be made in the process of teaching and learning science Davidson et al., (1994) states that metacognitive strategy may help students solve problems through effective planning, and involve the process of recognizing and classifying problems that should be solved and understand effective strategies to solve problems.

Metacognitive strategies had been employed especially in the field of mathematics and of technology of education (Claudia, 2000). Some studies on the implementation of metacognitive strategies possess an implementation of low metacognitive strategies (Claudia 2000). Competence acquired on the basis of metacognitive strategies employed to solve problems may improve students' confidence in solving other problems (Zimmerman and Rappaport, 1988).

Baird and Mitchell (1986) and Georghiades (2000) find that a group of teaching materials taught using metacognitive strategies may result in better achievement than the control group. Ability to control one's cognitive process has also been related to intelligence (Borkowski and Muthukrishna, 1992; Brown, 1987). Metakognitive enables students to get benefits of teaching (Carr et al., 1989) and influence the use and empowerment of cognitive strategies.

Borkowski and Muthukrishna (1992) states that metacognitive theories have a great potential to help teacher when they try to build a class environment that focuses on the creation of flexible and creative strategic learning activities. Meanwhile Paris and Winograd (1990) state that students may improve their learning if they are aware of their thinkings when they are reading, writing and solving problems at school. Teachers may encourage the birth of this awareness directly by giving the students information on strategies to solve problems effectively and discussess cognitive characteristics,and motivation to think.

In general, cognitive theories focuses attention to: (1) the roles of awarenes and arrangement of process to think; (2) different opinions on self evaluation and arrangement of cognitive development; (3) knowledge and main ability develop through experiences; and (4) constructive and strategic thinking (Paris and Winograd, 1990).

Swanson (1990) in his study shows that students in a high metacognitive stage is more competent in solving problems than those in a lower metacognitive stage. Another study shows that students who have good metacognitive skilsl in self-evaluation and are always concerned with their own effortsare mro strategic and may have better achievement than other students

Saad (2006) in their study on students' perception of a metacognitive approach divides its implementation into two phases. The phase one was done to know fourth year students' perception of the metacognitive stage using an instrument developed by O'Neil and Abedi (1996). Phase two was made through paired interviews using an instrumen of algebraic solution and interview guide. The sample of this study consisted of 238 fourth year students in ten elementary schools. The results showed that the fourth year students using a metacognitive approach when solvil addition problems were satisfactory with the average scores of 2.77. Bright students had positive perception when solving the problems than low or medium achievers.

In developing a teaching and learning model that may improve metacognitive capability, Nurdin (2007) showed such a model may give an impact on students' achievement (minimal 85% students got 6,5 scores or more). This result support metacognitive theories, that students' metacognitive capability may give a significant implication for one's learning success. It is a specific result since it is the aspect that is a characteristic that distinguishes the developed model from existing mathematics teaching-learning models. This result is in line with Mohamad (2000) statement that teaching cognitive strategies (metacognitive) may lead to the improvement of learning achievement.

Mustamin and Mikarna (2009) study metacognitive activities to solve formal and contextual mathematic problems. In learning mathematics, a metacognitive involvement is an important factor that should be considered in order to get a proper solution. Their results showed that metacognitive activities made in solving contextual and formal mathematic problems showed some differences, especially in the reflection aspect.

3. Metodhology

In this research, a quantitative and qualitative approach was employed and the subject was 43 students of 8 grade at State Junior High School 10 in Malang, Indonesia The data were obtained through an instrument developed by O'Neil and Abedi (1996) and completed with interviews. The data were then analysed in terms of average, frequency and percentage and then they were combined with the analysis of interviews. The interviewes were made in pairs, and there were 6 pairs of students with low, average and high achiements. So each category consisted of two pairs of students.

4. Findings and Discussions

Students' metacognitive approach consisting of four aspects were described using a descirptive quantitative analysis, namely frequency and percentage for each test item from the four metacognitive aspects. The first aspect deals with awarenes, and the results of the present research is presented in Table 1.

Based on Table 1, it was known that the average lowest response was 2.72 dealing with students' awareness on their ways of thinking. This information showed that the concerned items had the average lowest respons compared with other items. This statement was supported by a majority of response in a good category, with the frequency and percentage of 25 (58.14%), whereas in the average category, of 12 (27.91%). This means that test items related to students' awarenes of the way of thinking may be included into a good category, although the average of the items is 2.72.

The next is students' awareness of understanding problems before trying to solve them, with the average scoresof 2.88. The majority of students' responses to these items may be classified into a good category, with the frequency and percentage of 16 (37.21%), and the average category, 16)27.91%). This condition shows that students'awarenes in understanding problems before trying to solve problems tends to be in the medium category, namely between good and averafe, although the items may be included into a good category.

Students' awarenes before making use of their tahoughts to solbe problems and awarenes to arrangen their actions before trying to solve problems showed the same scores, namely 2.95. The majority of responses to the two items belongs to a good category, with the frequency and percentage of 21 (48.83%). Responses that come into average and good categories showed almost the same average namely 10.5 with the percentage of 24.42%. As other items, the average of the two items is higher from 2 but less than 3. This condition shows that the items may be included into a good category.

In general, the average frequency and percentage of students who showed awareness when understanding and solving problems were 30.4 (70.70%) and there were just 12.6 (29.30%) students who showed less awareness of the problems they wanted to solve. With the highest score of 3.05 and the lowest one of 2.72 and the average responses as a whole of 2.91, it can be stated that the average responses may be categorised into good, meaning that in general students were aware of their ways of thinking when they solved problems.

These results are supported by interviews with students, informing that students, the good or the average, had the awareness aspect when solving algebraic problems. As shown in the interviews, steps they made are as follows:

> ,… finding the side that has not been known yet, the one with no variable, for example, this is y and that one is z." (P1/T1/21-22).

This shows that students were aware that they should find a component that had not been known, namely giving variable y and z. Moreover, to solve the first problems, they said:

> .. finding the value od z .... equivalent to... (x +4) subtracted with (x+1) equals to x substracted x added 4 substracted 1 equals to 3… now we are finding the circumference, …(2x+3) added with (x+1) added with (x+4) addedd with (x+1) added with 3 added with (x + 2) (together with another student "(P1/T1/32-38).

Based on the transcripts of the interviews, it is suggested that students had some awareness of how to find the circumference of the known flat form. The understood that the circumference of a known flat model would be obtained though addition of sides forming the form of which the cinrcumferense is attempted to find. Moreover, it is also stated that students, in solving problems, had made use of an advanced thinking procedure, namely solving a problem from what they know first, then, thinking about various ways to come to something to be asked for, even by attemping it. There were some steps made, namely: (1) identifying something known and something to asked for in the problem; (2) thinking of a formula or the way that may relate the known thing and the thing to be asked and; (3) choosing a formula or an effective wat to solve the problem. The understanding of the result would be complete if the problem attempted intended to solve is as a material from the interviews as shown in Figure 1.

The result of the interview with students is as follows:

> "the side that is not known is called y and z, where y equals to 2x+3 substracted x+1 while *z* is equivalent to x+4 substracted x+1" (P1/T1/23-25).

This condition shows that students were aware of what to do before solving the problem on the circumference from the concerned graphic. Steps they should take to get the circumference and the width of the flat form were to find two unknown sides, as one of the students said below:

> …the side with unknown variable, take for example this y and that one z" (P1/T1/21-22). The next steps to fo to get the circumference and the with of the form, 'now we find the cinrcumference ..,… (2x+3) added (x+1) added (x+4) added (x+1) added 3 added (x+2)" (P1/T1/35-38).

The result shows that students were aware of the need to plan any action to take and also to understand what to do before solving the solution. But the students with average ability level showed less metacognitive awarenes in understanding the solution of problems.

The following is presented a strategic cognitive aspect from metacognitiion consisting of 5 items hat describe som items dealing with the understanding subjects and also strategies the student may implement to solve problems. The results of study obtained from the instrument the students had filled in are shown in Table 2.

Table 2 shows that the lowest average scores were 2.8.5 in terms of the strategies in asking to him-self on how is the interrelationship between tasks he faced and what he had known. And the majority of responses were good with the frequency and percentage of 19 (44.19%). The ability to choose and to organize appropriate information used to solve problems was responses by 20 (46.5%) with the average of 2.88. The next is strategies to think of the meaning of problems before students started to answert the problem which were shown by the average scores of2.95. The majority of responses to this item was in a good category with the frequency and percentage of 21 (48.83%). For the responses in average and very good categories were in balanced, but for the low category, the response was 1 (2.3%). For the strategies to attempt main topics in solving problems showed the highest average scores in the strategic covnitive aspect, namely 3.16.

The average of the whole responses were 2.98, based on the category in the metacognitive approach, it can be stated that the average fo the five response coming from the instrument items belongs to a good category, meaning that in general students were aware of thinking strategies to solve problems This result is supported by the conversation made during interviwes with a student:

> … now, problem *b* determine the area in x, the form of the picture is L, meaning divided into two ... give points ... and divided into two areas .. it is the width of form 1 and this formtwo, maning the are of form I added with form 2" (P1/T1/47-52). Another student also said ".. fing the area ...the a is known already ... also the b ... it is cut here .. oke then multiplied with the area of the first form equals to 2x+3 multiplied with x+1…" (P2/T2/47-52).

On the basis of the transcrip of the interview, it is known that students had cognitive strategies in solving problems, namely by dividing the known graphic into two by giving points in the part that divides the area into two, as shown by Figure 2. but there is a little bitdefect when finding the area as shiwn in the transcript the responden said. But, the student explicitly stated that to find an area is by multiplying the sides of which the length have veen known in x, and instead of adding them.

On the basis of figure 2, it may be obverved that the flat form is divided into two areas by giving points to distinguish the first and second areas. This is supported by the following interview::

> ".. next,calculating the area, meaning divided with two ... so that there is area one and area two then coun the area, meaning divided with two... so that there is area one and two.. area of form one .... square ... meaning p times p means x + 1 multiplied with x + 2 … equals to (x + 1)(x + 2) equals to x^2 2x added x added 2… equals to x^2 +3x + 2"…(P4/T4/50-55).

It is shown that students possessed cognitive strategies to understand the problems and hwo to solve them. It is also stated that in solving problem, a deductive thinking procedure is adopted, namely solving a problem that is from something general in order to obtain something very specific. There are some steps to take, namely: (1) identifying something known and asked in the problem; (2) choosing a formula, characeristic, or general requirement from one principle relating to something known and asked for;and (3) substituing the something known into the formula in order to obtain the answer to the question. The third aspect is planning of which the results are shown in Table 3.

The aspect of planning from the metacognitive approach consists of 5 tes items (as the Table 3 shows) informing on students's responses to the concerned aspect, attempting to understand problems before solving them with the lowest average scores of 2.70. The students' responses were under the good category, namely 23 (53.5%), followed by the average category, namely 12 (27.91%). But, students'planning to underatang before attempting to solve them was undert a good cagetory.

In general, students possessed planning to solve problems, where the average frquency and percentage were 30 (69.77%) and there were just 13 (30.23%) students who didn't really aware of such a planning. Viewed from the average highest responses of 2.93 and the lowest ones of 2.70 and the whole responses were 2.83 as a whole, it can be stated that in average the responses were good, meaning that students in general were aware of making making planning on their mind when solving problems.

The action of perfecting the results of study employing the metacognitice approach was made through interviews with students. From the interviews some information on the planning made is shown in the following transcript:

> .. finding the circumference, by adding its sides …… x+1 added 2x+3 added x+4 added x+1 added b which is equivalent to 3 added a… x +2…" then the "x is collected... (2x+x+x+x+x) added (3+1+4+1+3+2)" and the next "..x is equalized … .and collected meaning 6x yeah to" is obtained (P2/T2/47-54). Another student alsi says "…yeah… described 2x(x+1) added 3(x+1)…$2x^2$ plus 2x plus 3x plus 3 equals to... $2x^2 + 5x + 3$, this is the area"(P1/T1/68-70).

This shows that before solving problems, students had had a planning of what to do. This is shown by the first line of the transcript that to find the area of a flat form, the form should be divided into two , and then each area should be found and the whole area is obtained by adding the areas of form 1 and 2. this also applies to the students with low category of ability as follows

> ..the area of a ... square fowm ….. means length times width , berarti x+1 multiplied with x+2 … equals to (x+1)(x+2) equivalent to x^2 added 2x added x added 2… equals to x^2+3x+2…"(P4/T4/53-55).

This shows that students actually, when solving problems, made used of a deductive thinking procedure, namely solving problems from something general in order to have something specific. There were some steps taken: (1) identifying something known and that asked in the problems; (2) choosing a formula, characteristics, or general requirements from a principle relating something known to what is asked in the problems; and (3) substituting the something known in the formula in order to get something asked for.

The next aspect is review, meaning that in solving a problem, review should be made to the obtained result. It is intended to confirm whether the obtained answer is correct or needs some correction in order to have the intended answer.

Table 4 shows the review aspect from metacognition consists of 4 items. The highest average response is 2.93 in a good categpry, and this is in line with the students' veview shen they worked their jobs and used their knowledge of the problems they could do. Other three items had the same average scores, namely 2.79 dealing with the awareness of observing the mistakes, guiding progresses, and if necessary, changing strategies, and examining accuracy when working on problems. The majority of responses to the three items is under a good category. The response to the awarenes of examining mistakes had frequency and percentage of 24 (55.8%) meanwhile the majority of responses to the awarenes of changing strategies when solving problems was in the frewuency and percentage of 20 (46.51%). And the frequency and percentage of the awarenes to examine the acuracy when solving problems were 25 (58.1%). From the conditions above it can be stated the the majority of sudents had a gocc perception of the review aspects when solving problems.

From the whole responses, concerning with the category, the majority (52.56%) was good, 28.60% very good, , and 28.60% average, and low. In the table, it is shown that the whole average scores were2.85% and this informs that the review aspect in solving problems was categorized into goof. The results of interviews confirmed the previous description, as shown by the following transcript of the an interview:

> "…. yeah.. because it should be multiplied, instead of added" (P1/T1/84). Other students also said "… it cannot be added ...this is and the other x, one withquadrate, the other no … so it cannot be summed" (P2/T2/84-86).

The students that is grouped into excellent stundes showed that the four aspects in the metacognition may be categorized into good student, but the those in average and low groups showed that in solving problems in terms of the awareness aspect, plan and review they were not as expected, meaning that the three aspects

come into an average category. This also applied to the review aspect, meanwhile, the cognitive strategy aspet confirmed the results of the study from the instrument.

In this present study, conceerning with students' metacognitive approach to solving the algebra problems, it is done using the instrumen as that of the Saad (2006) and was also continued to paired interviews to confirm the results of the study. From the instrument of the stages of the metacognitive approach, the average scores of the approach were made in terms of awareness. Cognitive strategies, planning and review.

The descriptions above suggest that the four aspects in the stages of the meetacognitive approach to solving algebra problems showed that in general students were aware of the way they thought of solcing problems. To be aware of the thinking strategies when solving problem and also in planning the ways of thinking in problem solution and students' review when solving problems comes into a good category.

This also applies to the results of the interviews made in pairs, that may be categorized into very good, good and low. The students in the very good group showed that the results of the interviews made in thefour metacognitive aspects categorized as good, but for thise in average and low groups, it can be stated that solving problems in awarenes, planning and review aspects seemed to less relevant as expected, meaning that the three aspects were in low category. But, one of the aspects, namely review, was in an average category, while the cognitive strategy aspect confirmed the results of the instrument.

In short, the present study shows some strenghts, if compared with previous studies, namely : (1) implementing the model being developed ; (2) study stages of metacognitive treatmens as the principles adopted in the study, and (3) being added with interviews to complete the results in the stages of students' metacognitive appoach to solving algebra problems.

Conclusion

The stages of metacognitive approach to solving algebra problems in general are in a good category, with the scores for awareness, cognitive strategies, planing and reviews aspects of 2.98, 2.91, 2.83 and 2.85, respectively, meanwhile as a whole the average score is 2.89, that is under a good category.

The aspect of an awareness of understanding problems has average score of 2.88 where the majority of respnses to this item belongs to a good category, with the frequency and percentage of 16 (37/21%), and to average category , 13 (30.23%),. This condition is higher than those belongas to a very good one, with the frequency and percentage of 12 (27.01%). This shows that the awareness of understanding problems before attempting to solve problems tends to go into the intermediate category, namely between good and average category, although this item belongs to the good category based on the metacognitive category.

The same average score, namely 2.95 is shown by awareness before using the student's thoughts to solve problems and the awareness to plan actions before attempting to solve problems. The majority of the items belong to a good category with the frequency which is almost the same of which both averages is 10.5 with the frequency of 24.2%.

As other items, the average of both items is higher than 2 , less than 3. This condition shows that the items is in a good category. In general, students with some awareness of understanding and solving problems, show the frequency and percentage of 30.4 (70.70%) and those with little awareness of the problems to solve is merely 12.6 (29.30%) from the highest average response of 2.05 and the lowest one of 2.72 and the average whole respones of 2.91, and on the basis of the table of the metacognitive approach category, it is said that the average response belongs to a good category. It means that in general, students are aware of their ways of thinking in solving problems.

Students' cognitive strategies dealing with a metacognitive strategy made be categorize into two, namely students with very good strategies in solving problems showing good and very good responses with the average frequency and percentage of 32.2 (74.88%) and 10.8 (25.12%). Those with less cognitive strategies showing less and average responses showed the percentage of 1.86%, while the majority of the responses come under the good category with the percentage of 50.70%. the lowest average score of 2.86 deals with strategies in asking to oneself on how the interelationship beween the task he has and what he has known and the majority of the responses belong to a good category with the frequency and the percentage of 19 (44.19%). On the basis of the metacognitiveapproach category, it can be stated that the

cognitive strategy is under the good category. The whole average scores is 2.98 and on the basis of the metacognitive strategy approach, it can be stated that the average score of the four types of responses from the items is in a good category, meaning that students in general are aware of their strategies of thinking in solving problems.

In general, students with planning in solving problems, show average frequency and percentage of 30 (69.77%) and there are 13 (30.23%) students who show less awareness of planning in solving problems where the higest response is 2.93 and the whole average respone is 2.83

The aspect of review of the metacognitive strategy shows the highest average score of 2.93 that belongs to under a good category. Students' awarenes of investigating mistakes shows responses with the frequency and percentage of 24 (55.8%), while the awareness of changing strategy in solving problems show the majority of responses with the frequency and percentage of 20 (46.51%). Moreover, the awareness of considering accuracy whne solving problems shows responses woth the frequency and percentage of 25 (58.1%). The majority of students have perceptions of good in terms of the review aspect in solving problems, and the whole responses, 52.56 belongs to good categoru, and 18.6% ro very good category, meanwhile 28.84%, average and less cones. The average whole scores are 2.85, and this informs that the students' review aspect in solving problems belong to a good category.

Interviews are also made to confirm the results of the study. For students in the bright groups, the four metacognitive aspect confirm the resultts, while those in less and average groups, the cognitive strategy aspect confirm the results, and other aspects can be said to be relatively good.

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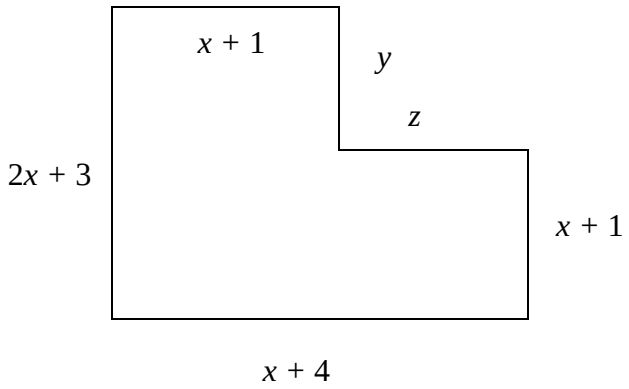
Figures

Figure 1: Problem for Paired Interviews

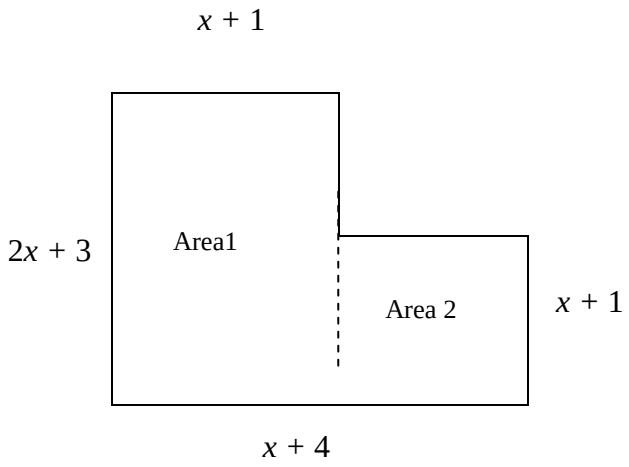


calculate:

a. The circumference of the flat form *x*

b. The area of flat form in *x*

Figure 2: Problem for Paired Interviews



Tables

Table 1: Response of Awareness of Metacognitive Strategies

Items	Responses				Average
	Low	Average	Good	Very Good	
I was aware of my own thinking	2 4.65%	12 27.91%	25 58.14%	4 9.30%	2.72
I was aware of which thinking technique or strategy to use and when to use it.	1 2.33%	11 25.58%	20 46.51%	11 25.58%	2.95
I was aware of the need to plan my course of action	1 2.33%	10 23.26%	22 51.16%	10 23.26%	2.95
I was aware of my ongoing thinking processes	1 2.33%	10 23.26%	18 41.86%	14 32.56%	3.05
I was aware of my trying to understand the test questions before I attempted to solve them	2 4.65%	13 30.23%	16 37.21%	12 27.91%	2.88
Average	1.4 3.29%	11.2 26.05%	20.2 46.98%	10.2 23.72%	2.91

Table 2: Response of Cognitive Strategies from Metacognitive Ones

Items	Responses				Average
	Low	Average	Good	Very Good	
I attempted to discover the main ideas in the test questions	0 0.0%	6 13.95%	24 55.81%	13 30.23%	3.16
I asked myself how the test questions related to what I already knew	1 2.3%	14 32.56%	19 44.19%	9 20.93%	2.86
I thought through the meaning of the test questions before I began to answer them	1 2.3%	11 25.58%	21 48.83%	10 23.26%	2.95
I used multiple thinking techniques or strategies to solve the test questions	0 0.0%	8 18.60%	25 58.14%	10 23.26%	3.05
I selected and organized relevant information to solve the test questions	2 4.6%	11 25.58%	20 46.51%	10 23.26%	2.88
Average	0.8 1.86%	10 23.26%	21.8 50.70%	10.4 24.19%	2.98

Table 3: Response of Planing of Metacognitive Strategies

Items	Responses				Average
	Low	Average	Good	Very Good	
I tried to understand the goals of the test questions before I attempted to answer	2 4.6%	9 20.93%	22 51.2%	10 23.26%	2.93
I tried to determine what the test required	1 2.3%	13 30.23%	20 46.51%	9 20.93%	2.86
I made sure I understood just what had to be done and how to do it	2 4.6%	10 23.26%	26 60.5%	5 11.63%	2.79
I determined how to solve the test questions	1 2.3%	12 27.91%	22 51.2%	8 18.60%	2.86
I tried to understand the test questions before I attempted to solve them.	3 7.0%	12 27.91%	23 53.5%	5 11.63%	2.70
Average	1.8 4.19%	11.2 26.05%	22.6 52.56%	7.4 17.21%	2.83

Table 4: Response of Review Aspect of Metacognitive Strategies

Items	Responses				Average
	Low	Average	Good	Very Good	
I checked my work while I was doing it	1 2.3%	11 25.58%	21 48.8%	10 23.26%	2.93
I corrected my errors	2 4.6%	11 25.58%	24 55.8%	6 13.95%	2.79
I almost always knew how much of the test I had left to complete	3 7.0%	7 16.28%	23 53.5%	10 23.26%	2.93
I kept track of my progress and, if necessary, I changed my techniques or strategies	4 9.30%	10 23.26%	20 46.51%	9 20.93%	2.79
I checked my accuracy as I progressed through the test	1 2.3%	12 27.91%	25 58.1%	5 11.63%	2.79
Average	2.2 5.12%	10.2 23.72%	22.6 52.56%	8 18.60%	2.85